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Formulating a Theoretical Model for Gravity on the Clarke-1880 Ellipsoid to Determine Practical Local-Geoid Models

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The determination of gravimetric geoid models using gravity anomalies requires the computation of normal gravity on specific ellipsoids. For global models, the International Gravity Formula is applied to the WGS 84 ellipsoid, while local models use the ellipsoid adopted for geodetic computation in the region of study. In countries that use the Clarke 1880 ellipsoid, a theoretical gravity model is computed on this surface to establish a precise local geoid model. This paper formulates a local theoretical gravity model for calculation of theoretical gravity values on the Clarke 1880 spheroid using the Somiglinana's closed formula for normal gravity, angular velocity, gravitational constant, and the Clarke 1880 spheroid parameters. The model is presented in two forms (A and B) and compared to ensure accuracy. Both forms of the model are suitable for accurate normal gravity computation on the Clarke 1880 ellipsoid.

Keywords: Clarke 1880, theoretical gravity, model, gravimetric, local geoid, ellipsoid.

1. Introduction

Gravity data has various applications, one of which is geoid modelling. Gravimeters or satellite observations are used to acquire this data. Regardless of the acquisition method, the data is typically reduced to the geoid. According to Eteje *et al.* [1], the geoid is a surface that would coincide with the surface of the oceans if they were free to adjust to the combined effects of the Earth's mass attraction and rotation's centrifugal force. It is used as a reference for a vertical coordinate system. To determine geoid models, the corresponding theoretical gravity of the observed points on a specified ellipsoid is required. Theoretical gravity is a latitude-dependent component of the ellipsoid and is obtained geodetically. The gravity anomaly is computed by finding the difference between the observed, as well as the absolute and theoretical gravity values.

The gravity anomalies observed at various points can either be categorized as global or local, depending on the ellipsoid from which they were obtained. When theoretical gravity values are computed using the WGS 84 (global) ellipsoid, they are considered global. However, if they are computed on the local ellipsoid adopted for geodetic computation in a particular country or region, these values are referred to as local. To calculate global gravity anomalies, the International Gravity Formula needs to be applied. [2] provides the International Gravity Formula as

$$g_T = 9.7803267714\left(\frac{1+0.00193185138639\sin^2\phi}{(1-0.00669437999013\sin^2\phi)^{1/2}}\right)\text{ms}^{-2} \quad (1)$$

Currently, there are no established methods for calculating normal or theoretical gravity on different local ellipsoids worldwide. To accurately determine local geoid measurements, it is crucial to compute theoretical gravity on the specific ellipsoid used for geodetic calculations within the study area. By computing normal gravity on the local ellipsoid, we can determine gravity anomalies of observed points in that particular region. In Nigeria, the Minna Datum uses the Clarke 1880 ellipsoid. While various local gravimetric geoid models have been determined across the country, the gravity anomalies of selected points were calculated using the International Gravity Formula, which is a global model. This was

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due to the absence of a local theoretical gravity model. Therefore, this paper formulates a local theoretical gravity model for calculation of theoretical gravity values on the Clarke 1880 spheroid using the Somiglinana's closed formula for normal gravity, angular velocity, gravitational constant, and the Clarke 1880 spheroid parameters.

To formulate the local theoretical gravity model on the Clarke 1880 ellipsoid, various parameters must be calculated. These include the semi-minor axis (*b*), first eccentricity squared (e^2), second eccentricity squared e'^2, , acceleration rate, *m*, q'_o, q_o theoretical gravity at the equator, g_e and theoretical gravity at the pole, g_P. The Clarke 1880 spheroid parameters, such as semi-major axis and flattening, along with other constants like angular velocity and gravitational constant are used to calculate the parameters.

1.1 Clarke 1880 Ellipsoid

The Clarke 1880 ellipsoid is a local ellipsoid used for geodetic calculations in Nigeria. Its semi-major axis (*a*) is 6378249.145m and its flattening (*f*) is 1/293.465. These values were provided by [3].

1.2 The Gravity Model

The International Gravity Formula on the WGS 84 ellipsoid was derived using Somiglinana's closed formula for normal gravity which is a commonly used general gravity formula. The formula (Equation, 2) was originally given by Moritz [4] and provides a method for calculating normal gravity.

$$\gamma = g_T = \frac{ag_e\cos^2\varphi + bg_p\sin^2\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}} \qquad (2)$$

Where,

$\gamma = g_T =$ theoretical gravity of a point

g_e = equatorial theoretical gravity

g_p = theoretical gravity at the pole

φ = observed point latitude

a = specified ellipsoid semi-major axis

b = specified ellipsoid semi-minor axis

The equation (2) can be expressed in a more convenient form (Equation, 3) for numerical computation, which is provided by Moritz [4] and Paraskevas *et al.* [5]. This information can be useful for further analysis and calculations.

$$g_T = \frac{g_e(1 + k\sin^2\varphi)}{(1 - e^2\sin^2\varphi)^{1/2}} \qquad (3)$$

[4] gave *k* in equation (3) as

$$k = \frac{bg_p}{ag_e} - 1 \qquad (4)$$

Where e^2 is the eccentricity squared of the specified ellipsoid.

To compute the value of k, it is necessary to calculate the theoretical gravity at both the equator and the pole. The computation of the theoretical gravity at these two locations is done according to the method described by Moritz [5].

$$g_e = \frac{GM}{ab}\left(1 - m - \frac{m}{6}.\frac{e'q'_o}{q_o}\right) \qquad (5)$$

$$g_p = \frac{GM}{a^2}\left(1 + \frac{m}{3}.\frac{e'q'_o}{q_o}\right) \qquad (6)$$

Where,

$$q'_o = 3\left(1 + \frac{1}{e'^2}\right).\left(1 - \frac{1}{e'}\arctan e'\right) - 1 \qquad (7)$$

$$2q_o = \left(1 + \frac{3}{e'^2}\right).\arctan e' - \frac{3}{e'} \qquad (8)$$

m = acceleration rate = $\dfrac{\omega^2 a^2 b}{GM}$ (9)

GM = gravitational constant = $3986005 \times 10^8\ \mathrm{m^3 s^{-2}}$

ω^2 = angular velocity = $7292115 \times 10^{-11}\ \mathrm{rad\ s^{-1}}$

To calculate the acceleration rate, *m*, q'_o and q_o, the first eccentricity squared, second eccentricity squared and the semi-minor axis of the specified spheroid are computed first. The formula to compute the semi-minor axis of an ellipsoid is given by Clynch [7] as

$$f = \frac{a - b}{a} \qquad (10)$$

$$\Rightarrow b = a(1 - f) \qquad (11)$$

and the first eccentricity squared according to Li and Gotze, [8] is given as

$$e^2 = \frac{a^2 - b^2}{a^2} \qquad (12)$$

[5] gave the second eccentricity squared as

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$$e'^2 = \frac{a^2 - b^2}{b^2} = \frac{e^2}{(1 - e^2)} \quad (13)$$

Where,

f = flattening of the spheroid

e = first eccentricity of the spheroid

e'^2 = second eccentricity squared of the spheroid

The conventional abbreviated series expansion for the gravity formula is given by Moritz [5] as

$$\gamma = g_T = g_e\left(1 + f^* \sin^2\varphi - \frac{1}{4} f_4 \sin^2 2\varphi\right) \quad (14)$$

Where, $f^* = \text{gravity flattening} = \frac{g_p - g_e}{g_e}$ (15)

$$f_4 = \frac{1}{2} f^2 + \frac{5}{2} fm \quad (16)$$

1.3 Gravity Anomaly

The gravity anomaly refers to the variation between the absolute gravity of a point reduced to the geoid and its corresponding latitude, in addition to the normal gravity calculated on a specific ellipsoid while also accounting for the effects of free air and rock. The gravity anomalies are categorized into two types: the free air gravity anomaly and the Bouguer gravity anomaly. The computation for the free air gravity anomaly (Δg_{fa}) is expressed as [2]:

$$\Delta g_{fa} = g_{Obs} - g_{fa} - g_o \quad (17)$$

Where, Δg_{fa} is free-air anomaly, g_{Obs} is observed gravity and g_o is theoretical gravity. g_{fa} is the free air gradient equal $0.3086[\text{mGa l/m}]\text{H}$ given in [13] and [14]. The Bouguer gravity anomaly, Δg_B is given by Vermeer [9] as

$$\Delta g_B = \Delta g_{fa} - 0.1119\,H \quad (18)$$

The determination of the orthometric heights for the points is possible through the utilization of the Digital Terrain Model available for the region.

1.4 Gravimetric Geoid

The gravimetric method for determining the geoid can be accomplished through the use of the widely used Stokes-integral, Equation (19), along with precise absolute gravity data as described in the works of [10], [11], and [12].

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} \Delta g S(\psi) d\sigma \quad (19)$$

Where,

N = geoid undulation

Δg = gravity anomaly

$S(\psi)$ = stokes function

γ = reference ellipsoid normal gravity

R = mean radius of the earth.

2. Methodology

2.1 Formulation of the Clarke 1880 Theoretical Gravity Model

In order to formulate the Clarke 1880 theoretical gravity model, several computations must be made including the semi-minor axis, *b*, first eccentricity squared, e^2, second eccentricity squared, e'^2, acceleration rate, m, q'_o, q_o, theoretical gravity at the equator, g_e, theoretical gravity at the pole, g_P, k, and presenting the final form of the model. Figure 1 illustrates the computation procedures in a flow chart.

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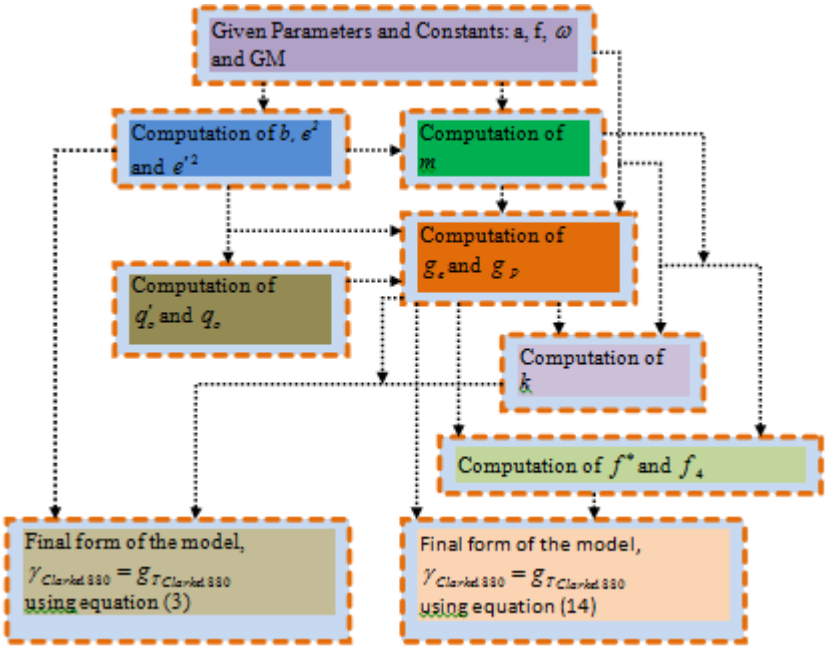
Given Parameters and Constants: a, f, ω and GM
Computation of b, e² and e'²
Computation of m
Computation of g_e and g_p
Computation of q'_o and q_o
Computation of k
Computation of f* and f_4
Final form of the model, γ_Clarke 880 = g_T Clarke 880 using equation (3)
Final form of the model, γ_Clarke 880 = g_T Clarke 880 using equation (14)

Figure 1: Computation procedures

The computations were done utilizing the provided semi-major axis and flattening of the Clarke 1880 ellipsoid, along with the angular velocity and gravitational constant.

2.1 Calculation of semi-minor axis, *b*

The semi-minor axis was calculated using equation (11) as

$$b = 6378249.145\left(1-\frac{1}{293.465}\right)$$
$$= 6356514.869\text{m}$$

2.2 Calculation of first eccentricity squared, e^2

The first eccentricity squared was computed using equation (12) as

$$e^2 = \frac{6378249.145^2 - 6356514.869^2}{6378249.145^2}$$
$$= 0.00680351145465242\text{m}$$

2.4 Calculation of second eccentricity squared, e'^2

The second eccentricity squared was calculated with equation (13) as

$$e'^2 = \frac{6378249.145^2 - 6356514.869^2}{6356514.869^2}$$
$$= 0.006850115\text{m}$$

2.5 Calculation of acceleration rate, *m*

The acceleration rate, *m* was calculated using equation (9) as

$$m = \frac{(7292115\times10^{-11})^2 \times 6378249.145^2 \times 6356514.869}{3986005\times10^8} = 0.003449778$$

2.3 Calculation of q'_o

With equation (7), q'_o was calculated as

$$q'_o = 3\times\left(1+\frac{1}{0.006850115}\right)\times\left(1-\frac{1}{0.082765427}\times\arctan 0.082765427\right)-1$$
$$= 0.002732032$$

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2.6 Calculation of q_o

Using equation (8), q_o was calculated as

$$2q_o = \left(1 + \frac{3}{0.006850115}\right) \times \arctan 0.082765427 - \frac{3}{0.082765427} = 0.000150304$$

$$\therefore q_o = \frac{0.000150304}{2}$$

$$= 0.000075152$$

The value of $\arctan 0.08276542\ 7$ was converted to radian before it was used for the computation.

2.7 Calculation of theoretical gravity at the equator, g_e

The theoretical gravity at the equator on the Clarke 1880 ellipsoid, g_e was calculated using equation (5) as

$$g_e = \frac{3986005 \times 10^8}{6378249.145 \times 6356514.869}\left(\begin{array}{l}1 - 0.003449778 \\ \quad - \dfrac{0.003449778}{6} \times \dfrac{0.082765427 \times 0.002732032}{0.000075152}\end{array}\right)$$

$$= 9.780519381 \text{ms}^{-2}$$

2.8 Calculation of theoretical gravity at the pole, g_P

With equation (6), the theoretical gravity at the pole, g_p was calculated as

$$g_p = \frac{3986005 \times 10^8}{6378249.145^2}\left(1 + \frac{0.003449778}{3} \times \frac{0.082765427 \times 0.002732032}{0.000075152}\right)$$

$$= 9.831842301 \text{ms}^{-2}$$

2.9 Calculation of k

By substituting the obtained values of g_e and g_P into equation (4), *k* was calculated as

$$k = \frac{6356514.869 \times 9.831842301}{6378249.145 \times 9.780519381} - 1$$

$$= 0.00182202113732433$$

2.10 Final forms of the formulated Clarke 1880 spheroid theoretical gravity model, $\gamma_{Clarke1880} = g_{T\,Clarke1880}$

Similarly, by putting the values of g_e, *k,* and e^2 into equation (3), the Clarke 1880 ellipsoid theoretical gravity formula, that is, model A is presented as

$$\gamma_{Clarke1880(A)} = g_{T\,Clarke1880(A)} = 9.780519381\left[\frac{(1 + 0.00182202113732433\sin^2\phi)}{(1 - 0.00680351145465242\sin^2\phi)^{1/2}}\right]\text{ms}^{-2}$$

The series expansion formula given in equations (14) to (16) was used to obtain the series expansion form of model B as follows:

f^* was calculated using equation (15) as

$$f^* = \frac{9.831842301 - 9.780519381}{9.780519381}$$

$$= 0.00524746365716546$$

while f_4 was calculated with equation (16) as

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$$f_4 = \frac{1}{2}\times 0.00340756137869933^2 + \frac{5}{2}0.00340756137869933\times 0.003449778$$
$$= 0.00003519406296951$$

Substituting the calculated values of f^* and f_4 into equation (14), we obtained

$$\gamma = g_T = 9.780519381\left(1+0.00524746365716546\sin^2\varphi - \frac{1}{4}\times 0.00003519406296951\sin^2 2\varphi\right)\text{ms}^{-2}$$
$$= 9.780519381\left(1+0.00524746365716546\sin^2\varphi - 0.0000087985157424\sin^2 2\varphi\right)\text{ms}^{-2}$$

Hence, the Clarke 1880 spheroid theoretical gravity model series expansion form, that is, model B is

$$\gamma_{Clarke1880\,(B)} = g_{T\,Clarke1880\,(B)} = 9.780519381\left(1+0.00524746\ 36\sin^2\varphi - 0.00000879\ 85\sin^2 2\varphi\right)\text{ms}^{-2}$$

3. Results

3.1 Comparison of the Models

The comparison of the two model forms is shown in Table 1 and Figures 2 and 3. Model A, obtained with equation (3), and Model B, obtained with equation (14) using the series expansion formula, were evaluated for their suitability and agreement. The theoretical gravity values of six points were calculated using Model A and Model B, based on their latitudes. The normal gravity of the points was then compared using both model forms to determine any differences. Table 1 reveals that the normal gravity of the six points obtained from Model A is identical to that of Model B, indicating no difference in the computation results of the two model forms. It implies that any of the models forms can the applied to a region where geodetic computation is done using the Clarke 1880 spheroid. Figures 2 and 3 also demonstrate the agreement and suitability of both model forms, as the plotted normal gravity from both forms have the same shape and differences.

Table 1: Comparison of the models

Latitude	Model A (Equation (3))	Model B (Equation (14))	Difference
6.331514607	9.781140782	9.781139430	0.000001352
6.301664360	9.781134960	9.781133620	0.000001339
6.287544686	9.781132215	9.781130881	0.000001334
6.257952119	9.781126483	9.781125161	0.000001321
6.237847990	9.781122603	9.781121290	0.000001313
6.214210013	9.781118058	9.781116755	0.000001303

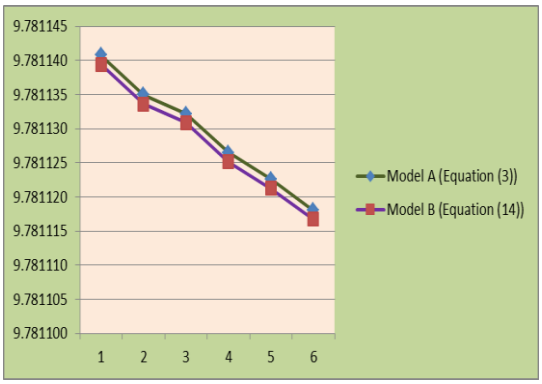

Figure 2: Computed normal gravity

Figure 3: Computed normal gravity differences

4. Conclusion

In order to determine geoid models using the gravimetric method, gravity anomalies of specific

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points must be used. To do this, normal gravity values of points with known absolute or observed gravity values must be calculated. The establishment of acceptable local geoid model required that the calculation of the theoretical gravity values is done on the spheroid used for geodetic computation in the area of study. In this study, a local theoretical gravity model was formulated on the Clarke 1880 ellipsoid, which is used for geodetic computation in some countries like Nigeria. The derived model was presented in two forms (model A and model B) and compared for suitability and agreement. The results showed that both forms of the model were in agreement and can be used for accurate normal gravity computation in regions/countries where the Clarke 1880 ellipsoid is used as a reference surface for geodetic calculation.

Conflict of Interest

The authors declare no conflict of interest.

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